Axial force-Moment interaction in the LARSA hysteretic beam element

This document briefly discusses the modeling of tri-axial interaction (i.e. between axial force and bending moments) in the LARSA beam element. A cantilever beam made of the S3x5.7 cross-section is used as an example.

![Moment Curvature Curves for different Axial Force Levels](image1)

**Figure 1. Moment Curvature Curves for different Axial Force Levels ($p = P/P_y$)**

![PM-Interaction Diagram](image2)

**Figure 2. PM-Interaction Diagram**
Figure 1 shows the moment-curvature curves of the S3x5.7 cross-section (as an example) at different levels of axial force. The symbol $p$ denotes the ratio of axial force level to the yield axial force at zero bending moment ($P/P_y$). The “yield moments” for the different levels of axial force are obtained from these curves as the intersection of the initial elastic line and the post-yield asymptote and are plotted as the triangles in Figure 2. Similar to $p= P/P_y$, $m= M/M_y$. The PM interaction diagram is obtained by fitting a curve through these points. There are several possibilities for this curve-fitting. For example, a curve of the form $\Phi = |p|^{b_1} + b_2 + |m| - 1 = 0$ with $b_1 = 1.5$ and $b_2 = -0.3$ can be used as done by Sivaselvan and Reinhorn (2002). Another option is to use the generic curve for wide-flanged steel sections, given by $\Phi = p^2 + m^2 + 3.5p^2m^2 - 1 = 0$. This is the specialization to two dimensions of the equation $p^2 + m^2 + 3.5p^2m^2 + 3p^2m^2 + 4.5m^2m^2 = 1$ proposed by McGuire (2000). This yield surface is built into the hysteretic beam element in LARSA. A further possibility is to use a piecewise linear curve fit with the two lines $p = -7.004m + 7.004$ and $p = -0.8708m + 1$. The hysteretic beam element in LARSA also allows the option of using such a piecewise linear yield surface. All three possibilities are shown in Figure 2.

The piecewise linear yield surface is specified as the intersection of several planes of the standard form, $ax + by + cz = d$, where $x=M_x$, the bending moment about the strong axis, $y=M_y$, the bending moment about the weak axis and $z=P$, the axial force. $a$, $b$ and $c$ are components of the unit vector normal to the plane and $d$ is the perpendicular distance of the plane from the origin. In our example, we consider only interaction between axial force and the strong axis bending moment. The set of planes therefore consists of the two planes, $7.004m_x + 0m_y + p = 7.004$ and $0.8708m_x + 0m_y + p = 1$, their counterparts in the other three quadrants of the P-M interaction diagram and the two planes denoting the limits of the weak axis bending moment, $0m_x + m_y + 0p = 1$ and $0m_x - m_y + 0p = 1$. The ten interaction planes, shown in Figure 3, are:

1. $m_z + 0m_y + 0.142776p = 1$
2. $0.8708m_z + 0m_y + p = 1$
3. $-m_z + 0m_y + 0.142776p = 1$
4. $-0.8708m_z + 0m_y + p = 1$
5. $-m_z + 0m_y - 0.142776p = 1$
6. $-0.8708m_z + 0m_y - p = 1$
7. $m_z + 0m_y - 0.142776p = 1$
8. $0.8708m_z + 0m_y - p = 1$
9. $0m_z + m_y + 0p = 1$
10. $0m_z - m_y + 0p = 1$

![Figure 3. P-M interaction planes](image)

The equations of the planes are now written in terms of the absolute quantities, rather than the normalized quantities. For the cross-section under consideration, $P_y = 60.12$ kip, $M_x = 23.51$ kip-in and $M_y = 70.20$ kip-in. The equation of plane (1) then becomes:

$$\frac{M_x}{70.20} + 0\frac{M_y}{23.51} + 0.142776\frac{P}{60.12} = 1 \Rightarrow 0.014245M_x + 0M_y + 0.002375P = 1$$

(1a)
Similarly the equations of the other planes are:

\[0.012405M^x + 0M^y + 0.016633P = 1\] (2a)
\[-0.014245M^x + 0M^y + 0.002375P = 1\] (3a)
\[-0.012405M^x + 0M^y + 0.016633P = 1\] (4a)
\[-0.014245M^x + 0M^y - 0.002375P = 1\] (5a)
\[-0.014245M^x + 0M^y - 0.016633P = 1\] (6a)
\[0.014245M^x + 0M^y - 0.002375P = 1\] (7a)
\[0.012405M^x + 0M^y - 0.016633P = 1\] (8a)
\[0M^x + 0.042535M^y + 0P = 1\] (9a)
\[0M^x - 0.042535M^y + 0P = 1\] (10a)

These planes are input to the LARSA section properties as shown in Figure 4.

Figure 4. Yield surface input in LARSA

Figure 5 shows a cantilever beam with a support displacement applied in the x-direction at the top joint. Figure 6 shows the top displacement-shear force relationship for axial force levels corresponding to the 0%, 25% and 50% of the axial yield force, using (a) the built-in yield surface and (b) the piecewise linear yield surface of Figure 4. The negative slope seen in the latter two axial force levels is the result of \(P\Delta\) effect. The decrease in yield force (yield moment)
due to the presence of axial force can be observed. The difference between using the two yield surfaces results from the different modeling of post-yield behavior in the two cases.

Figure 5. Cantilever with top support displacement in x direction

Figure 6. Force-displacement curves of cantilever beam
Summary of Procedure

1. Using a section analysis tool, obtain the moment curvature curves at different levels of axial load.
2. For each of these curves, i.e. for each value of $P$, using a bilinear approximation find the value of the yield moment $M$.
3. We now have $(P,M)$ pairs. If the section analysis tool can directly generate these pairs, then the first two steps can be bypassed.
4. Normalize the $(P,M)$ pairs with $P_y$ and $M_y$, the axial yield force at zero bending moment and the yield bending moment at zero axial force respectively, to obtain $(p,m)$ pairs.
5. Plot these points on a graph as shown in Figure 2. This is the interaction diagram. Depending on the symmetry of the cross-section, one, two or all four quadrants of the interaction diagram may need to be considered. Since in the above example, the cross-section was symmetric and the material properties were identical in tension and compression, only the first quadrant was considered.
6. Fit a desired number of straight lines through these points. In Figure 2, two straight lines have been used for each quadrant of the interaction diagram.
7. List the equations of these lines, including those in the unconsidered quadrants as listed in (1)-(8).
8. Add two bounding planes for the weak axis bending moment as done in (9)-(10)
9. Replace $p$ by $P/P_y$ and $m$ by $M/M_y$ as listed in (1a)-(10a)
10. Enter the coefficients under LARSA section properties. Note that the coefficients need to have units of 1/Moment, 1/Moment and 1/Force respectively. The material unit system is used. For example, if the material force unit is kip and the material length unit is inches, then the coefficients would have units of 1/(kip-in), 1/(kip-in) and 1/kip respectively. The offset has no units.
11. Any nonlinear analysis would then use this piecewise linear yield surface for the cross-section.

References